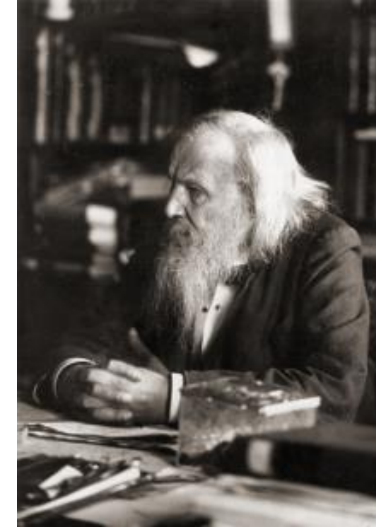
Topology and the Classification of Matter

New Physics Hidden in Plain Sight

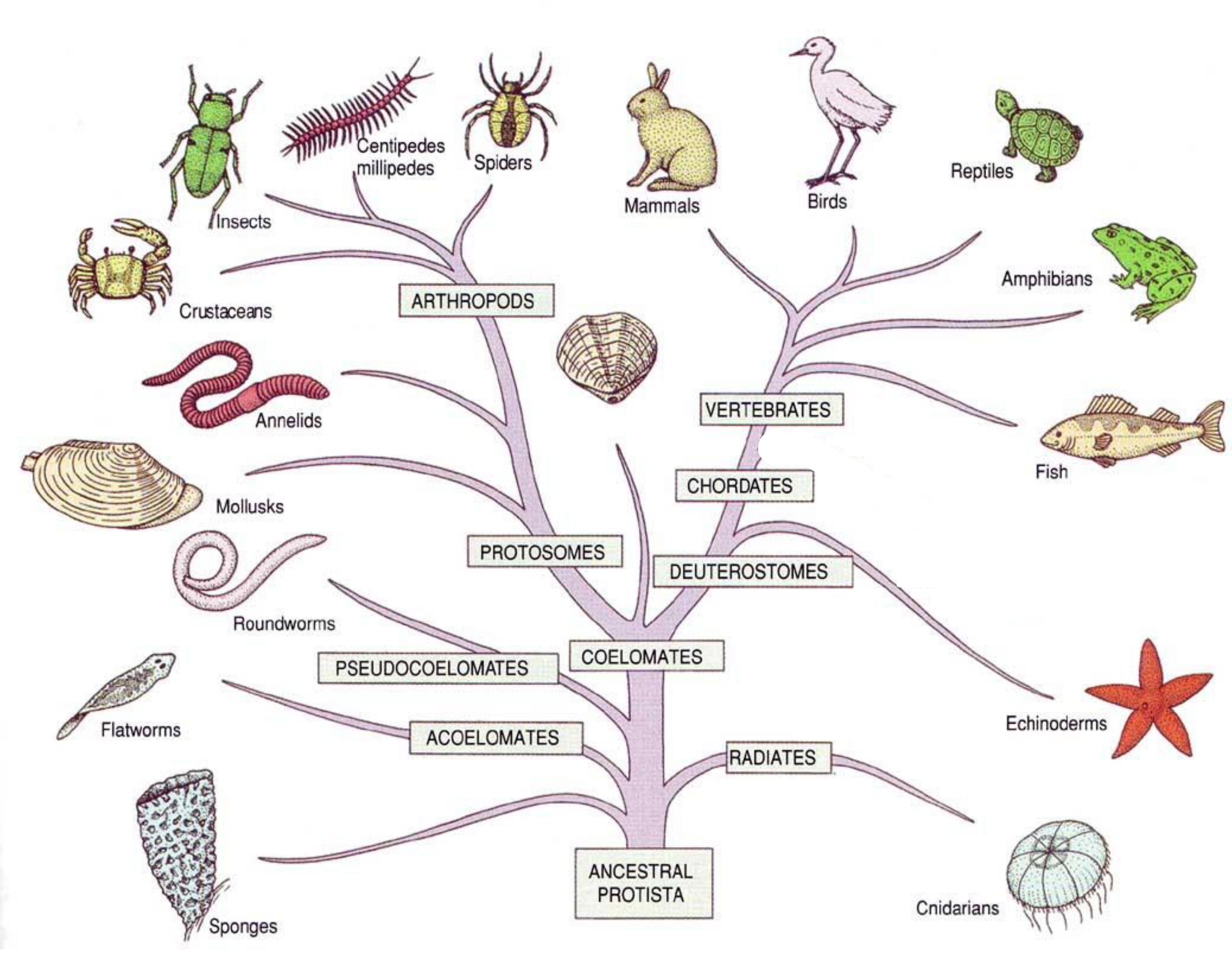
Mendeleev's Periodic Table (1869)

Arrange by Atomic Weight

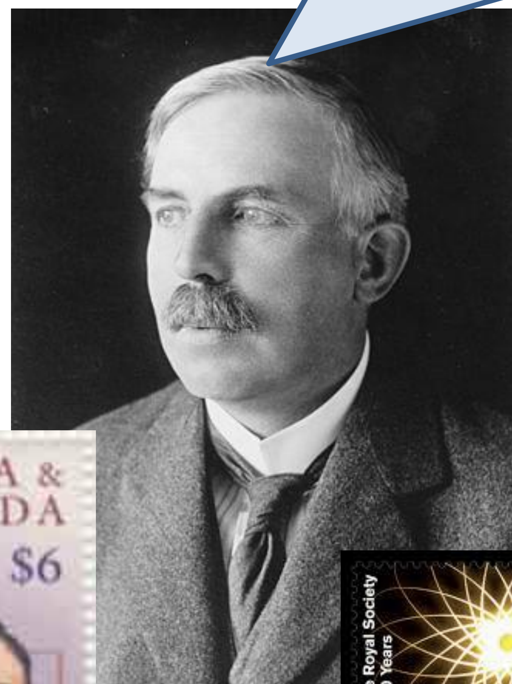


Reihen	Gruppe I. — R^2O	Gruppe II. — RO	Gruppe III. — R^2O^3	Gruppe IV. RH^4 RO^2	Gruppe V. RH^3 R^2O^5	Gruppe VI. RH^2 RO^3	Gruppe VII. RH R^2O^7	Gruppe VIII. — RO^4
1	H=1							
2	Li=7	Be=9,4	B=11	C=12	N=14	O=16	F=19	
3	Na=23	Mg=24	Al=27,3	Si=28	P=31	S=32	Cl=35,5	
4	K=39	Ca=40	—=44	Ti=48	V=51	Cr=52	Mn=55	Fe=56, Co=59, Ni=59, Cu=63.
5	(Cu=63)	Zn=65	—=68	—=72	As=75	Se=78	Br=80	
6	Rb=85	Sr=87	?Yt=88	Zr=90	Nb=94	Mo=96	—=100	Ru=104, Rh=104, Pd=106, Ag=108.
7	(Ag=108)	Cd=112	In=113	Sn=118	Sb=122	Te=125	J=127	
8	Cs=133	Ba=137	?Di=138	?Ce=140	—	—	—	— — — —
9	(—)	—	—	—	—	—	—	
10	—	—	?Er=178	?La=180	Ta=182	W=184	—	Os=195, Ir=197, Pt=198, Au=199.
11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	—	—	
12	—	—	—	Th=231	—	U=240	—	— — — —

Phylogenetic Tree



All science is either physics or stamp collecting.

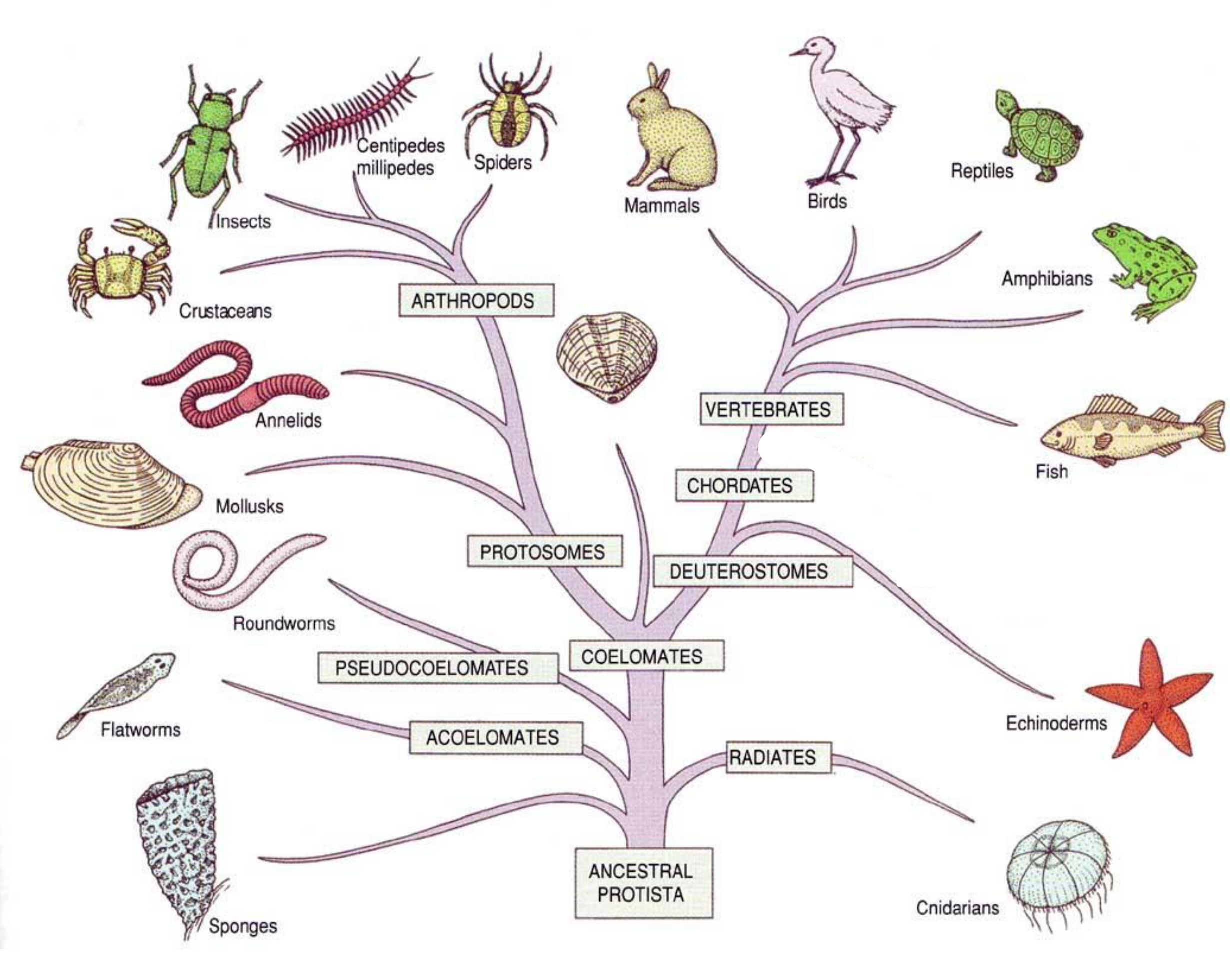


Ernest Rutherford

Nobel Prize 1908

... in Chemistry

Phylogenetic Tree



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11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	—	—	
12	—	—	—	Th=231	—	U=240	—	— — — —

He = 4
Ne = 20.2
Ar = 40
Kr = 83.8
Xe = 131

Henry Moseley, Oxford, 1913
Died 1915, Gallipoli, Age 27

X-ray technique "measures" number of protons in an atom

Put elements in (undisputable) correspondence with positive integers.

Isaac Asimov: His death might well have been the most costly single death of the War to mankind generally

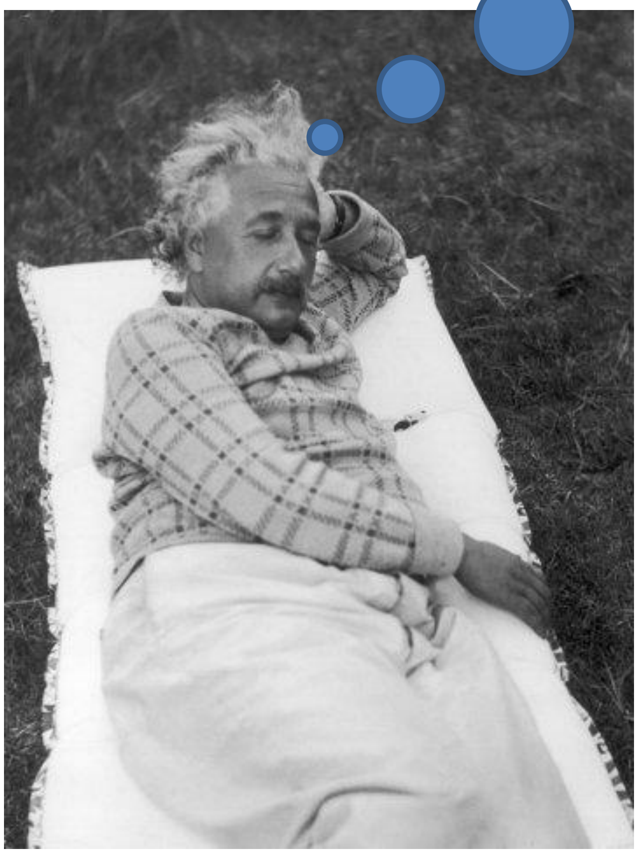
Robert Millikan: Had the European war had no other result than the snuffling out of this young life, that alone would make it one of the most hideous and most irreparable crimes in history."

Arrange by Number of Protons

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** 1.008																	2 **He** 4.0026
3 **Li** 6.94	4 **Be** 9.0122											5 **B** 10.81	6 **C** 12.011	7 **N** 14.007	8 **O** 15.999	9 **F** 18.998	10 **Ne** 20.180
11 **Na** 22.990	12 **Mg** 24.305											13 **Al** 26.982	14 **Si** 28.085	15 **P** 30.974	16 **S** 32.06	17 **Cl** 35.45	18 **Ar** 39.948
19 **K** 39.098	20 **Ca** 40.078	21 **Sc** 44.956	22 **Ti** 47.867	23 **V** 50.942	24 **Cr** 51.996	25 **Mn** 54.938	26 **Fe** 55.845	27 **Co** 58.933	28 **Ni** 58.693	29 **Cu** 63.546	30 **Zn** 65.38	31 **Ga** 69.723	32 **Ge** 72.630	33 **As** 74.922	34 **Se** 78.97	35 **Br** 79.904	36 **Kr** 83.798
37 **Rb** 85.468	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.224	41 **Nb** 92.906	42 **Mo** 95.95	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.91	46 **Pd** 106.42	47 **Ag** 107.87	48 **Cd** 112.41	49 **In** 114.82	50 **Sn** 118.71	51 **Sb** 121.76	52 **Te** 127.60	53 **I** 126.90	54 **Xe** 131.29
55 **Cs** 132.91	56 **Ba** 137.33	57-71 *	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.84	75 **Re** 186.21	76 **Os** 190.23	77 **Ir** 192.22	78 **Pt** 195.08	79 **Au** 196.97	80 **Hg** 200.59	81 **Tl** 204.38	82 **Pb** 207.2	83 **Bi** 208.98	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89-103 #	104 **Rf** (265)	105 **Db** (268)	106 **Sg** (271)	107 **Bh** (270)	108 **Hs** (277)	109 **Mt** (276)	110 **Ds** (281)	111 **Rg** (280)	112 **Cn** (285)	113 **Nh** (286)	114 **Fl** (289)	115 **Mc** (289)	116 **Lv** (293)	117 **Ts** (294)	118 **Og** (294)

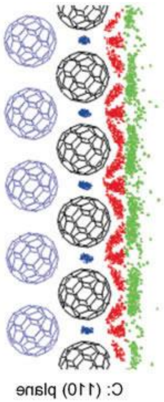
* Lanthanide series	57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140.91	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.96	64 **Gd** 157.25	65 **Tb** 158.93	66 **Dy** 162.50	67 **Ho** 164.93	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.05	71 **Lu** 174.97
# Actinide series	89 **Ac** (227)	90 **Th** 232.04	91 **Pa** 231.04	92 **U** 238.03	93 **Np** (237)	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (262)

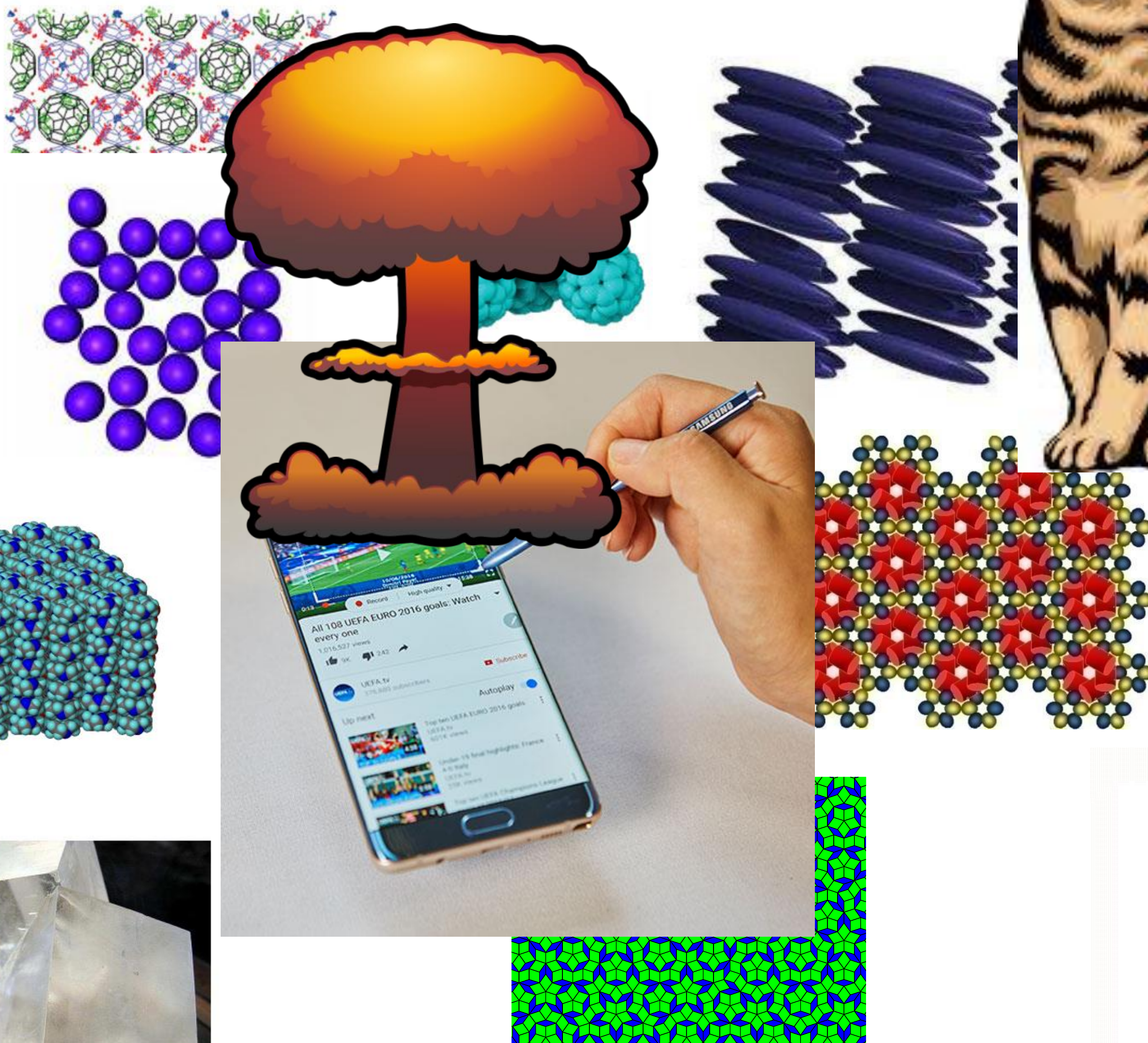
THE PERIODIC TABLE OF EVERYTHING



Classification and Organization
of
All Matter

All Matter




SAMSUNG
All 108 UEFA EURO 2016 goals: Watch every one
Autoplay
Up next

How Many Types of Matter

- ~100 Chemical Elements
- Over 30 Million Chemical Compounds are in "Chemical Abstract Services"
- Infinite number are possible in principle?
- Can form mixtures, alloys, solutions of compounds

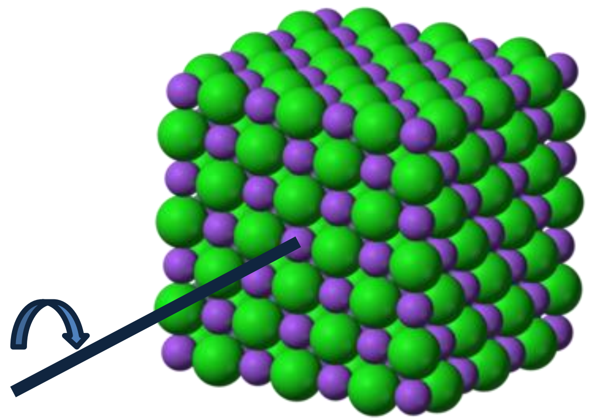
More possibilities than atoms in the universe!

Guiding Principles

Lev Landau 1908-1968
Nobel Prize 1962

CLASSIFY BY SYMMETRIES

Obvious symmetries



Rotate by 90 degrees
Get back same thing

Subtle (abstract) symmetries:

- Time Reversal, T
  (Materials without Magnetism have T symmetry)

- Charge Conjugation, C
  Replace all + charges with – charges
  Ex: electrons become positrons
  (Superconductors have C symmetry)

Guiding Principles

TOPOLOGY

The Nobel Prize in Physics 2016

David J. Thouless
Prize share: 1/2

F. Duncan M. Haldane
Prize share: 1/4

J. Michael Kosterlitz
Prize share: 1/4

DPhil Oxford 1969

Guiding Principles

TOPOLOGY

Wikipedia: Topology is concerned with the properties of space that
are preserved under continuous deformations, such as stretching and bending,
but not tearing or gluing.

Guiding Principles

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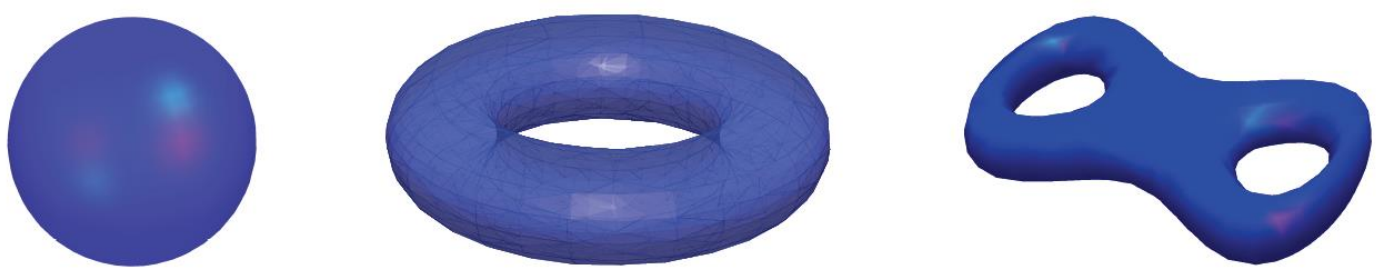
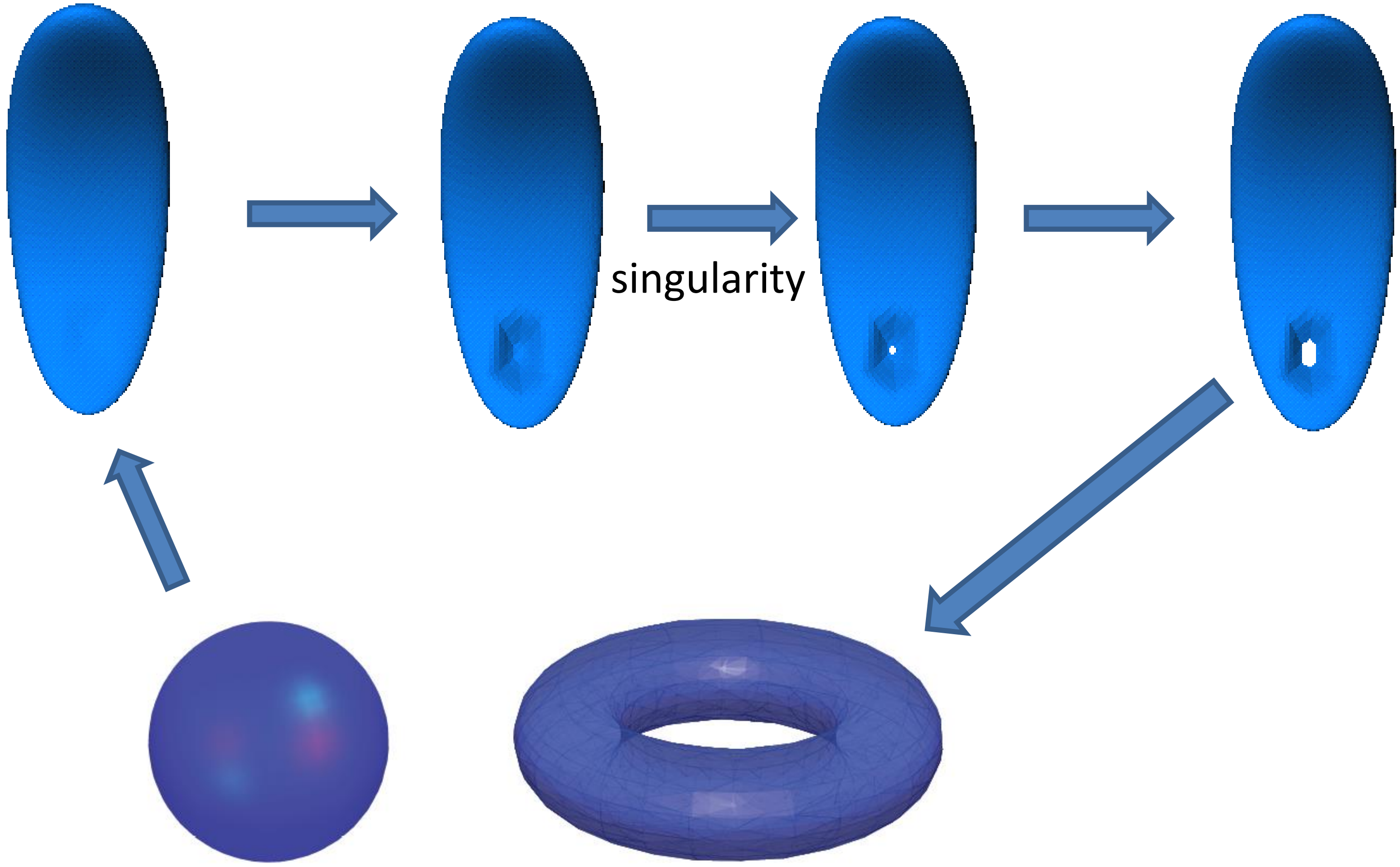


Cannot deform from torus to sphere without encountering a *"singularity"*



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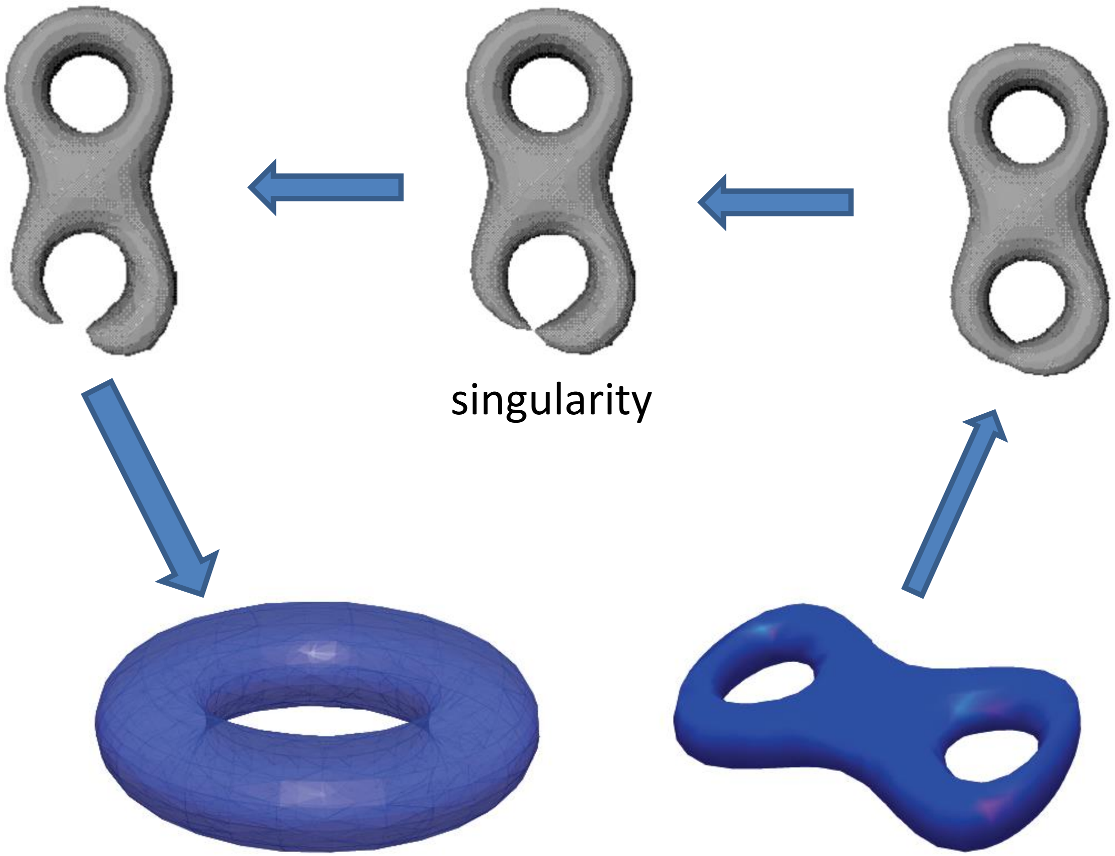
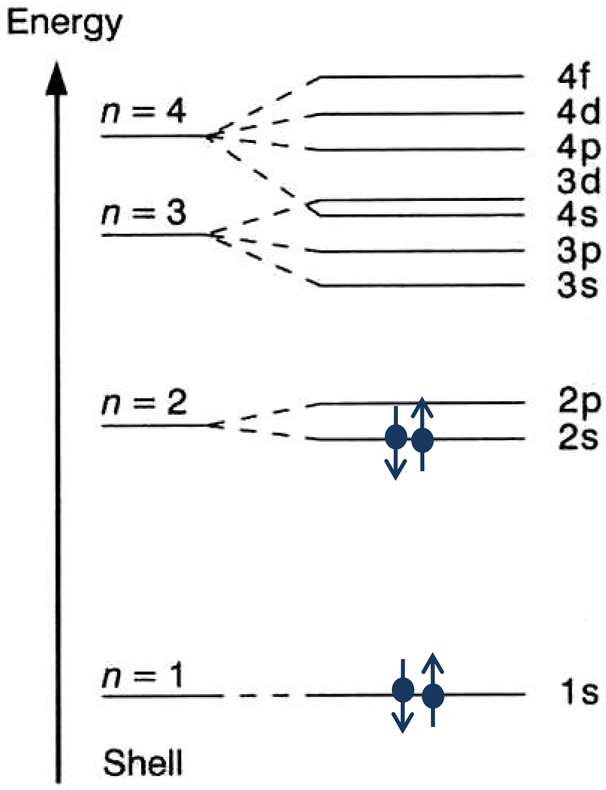


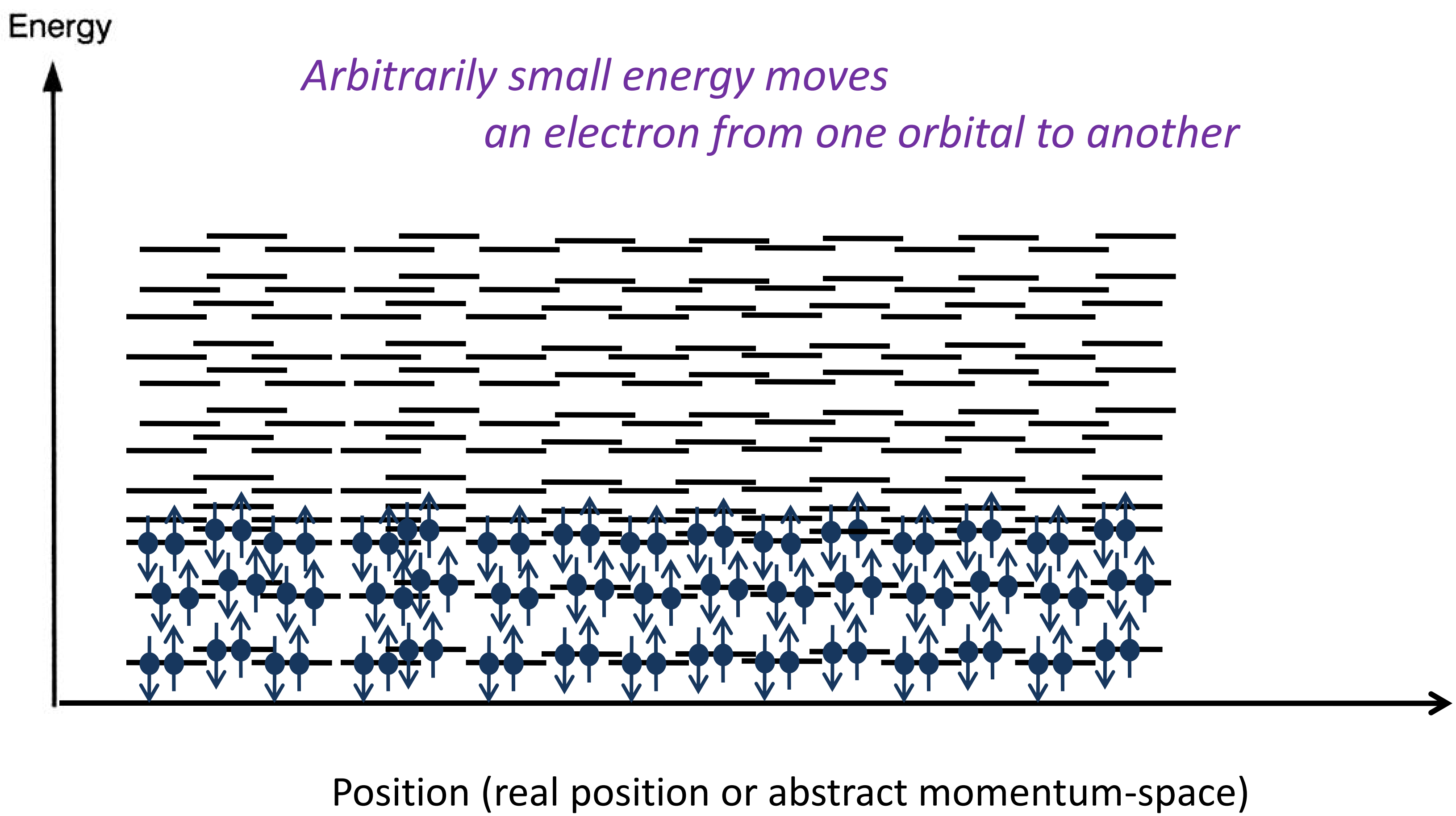
Cannot deform from torus to sphere without encountering a *"singularity"*

How does this help in classifying types of matter?

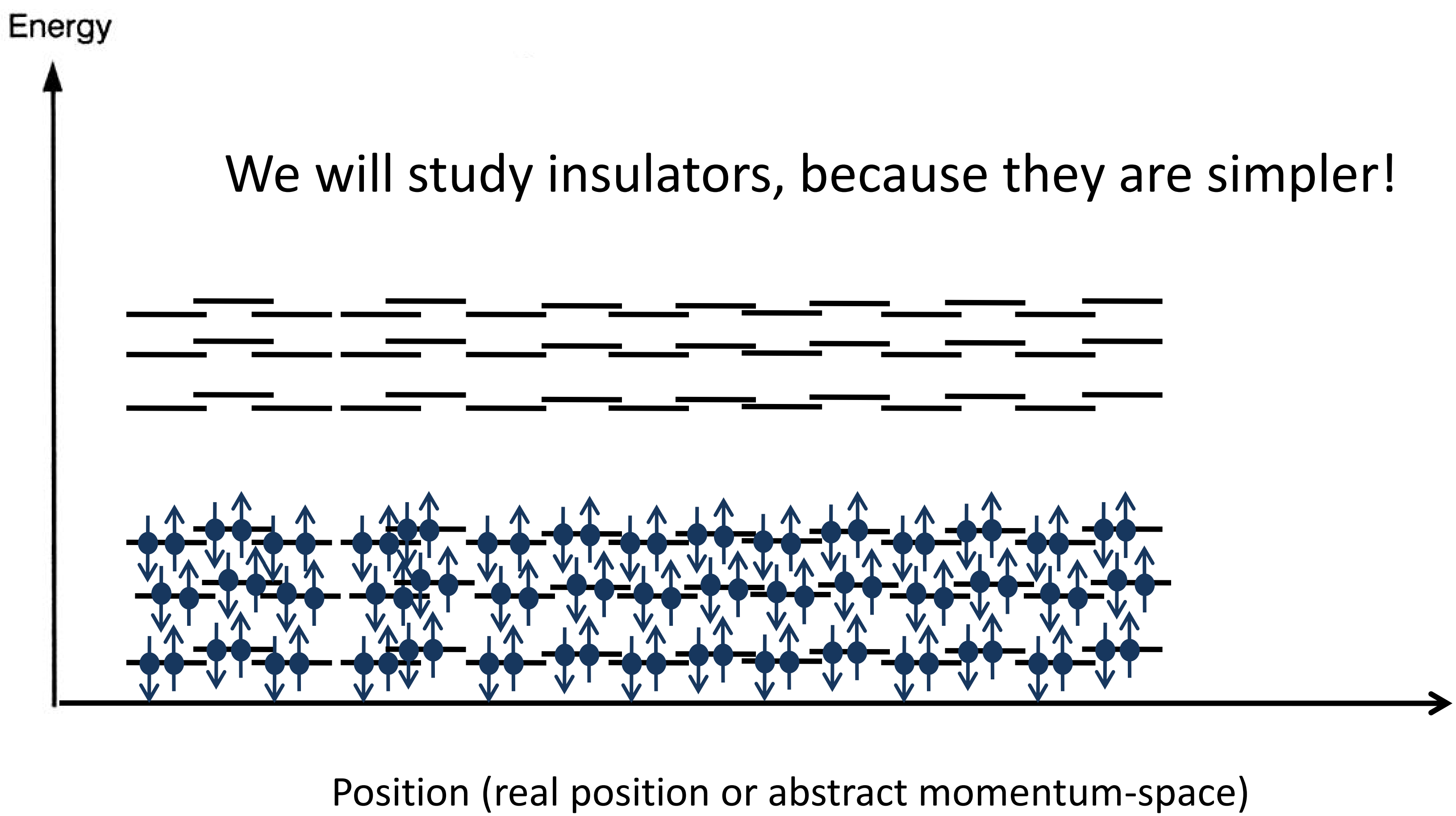
• Reminder: Atoms have energy levels that get filled with electrons



- Materials made of many atoms also have energy levels that get filled with electrons.
- If there are low energy excitations (empty states close to filled states), it is an electrical conductor.



- Materials made of many atoms also have energy levels that get filled with electrons.
- If there are low energy excitations (empty states close to filled states), it is an electrical conductor.
- If there is an energy gap, it is an insulator

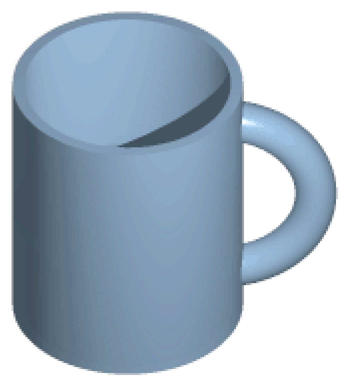


We will study insulators, because they are simpler!

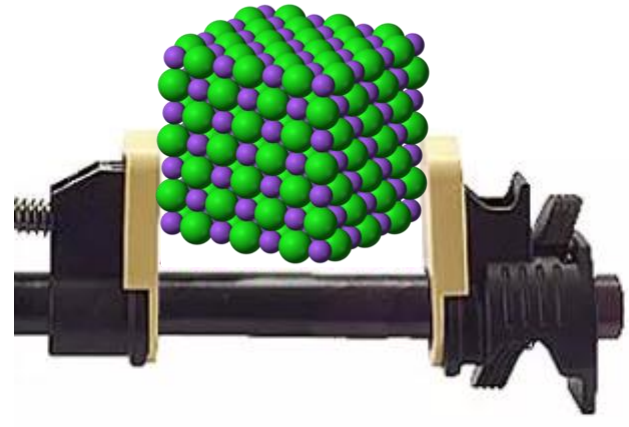
Guiding Principles

TOPOLOGY

Think about ***Deformations*** of the Insulator on the microscopic scale...



What Deformations can we do?



•Deformations that you could really do...
ex: Change the pressure
... add strain, apply electric fields, etc...

•Deformation that you could never really do...

ex: Coulomb interaction between nuclear charge Q and electron charge Q' is:

$$V(r) = \frac{QQ'}{4\pi\epsilon_0 r}$$

Imagine changing it to:

$$V(r) = \frac{QQ'}{4\pi\epsilon_\alpha\, r^\alpha}$$

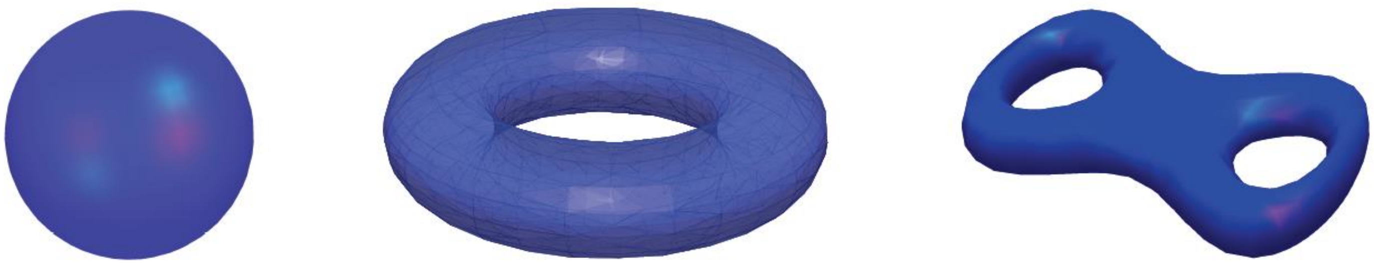
Imagine changing the mass of the electrons, protons, neutrons

Imagine changing the speed of light

etc...

Guiding Principles

TOPOLOGY



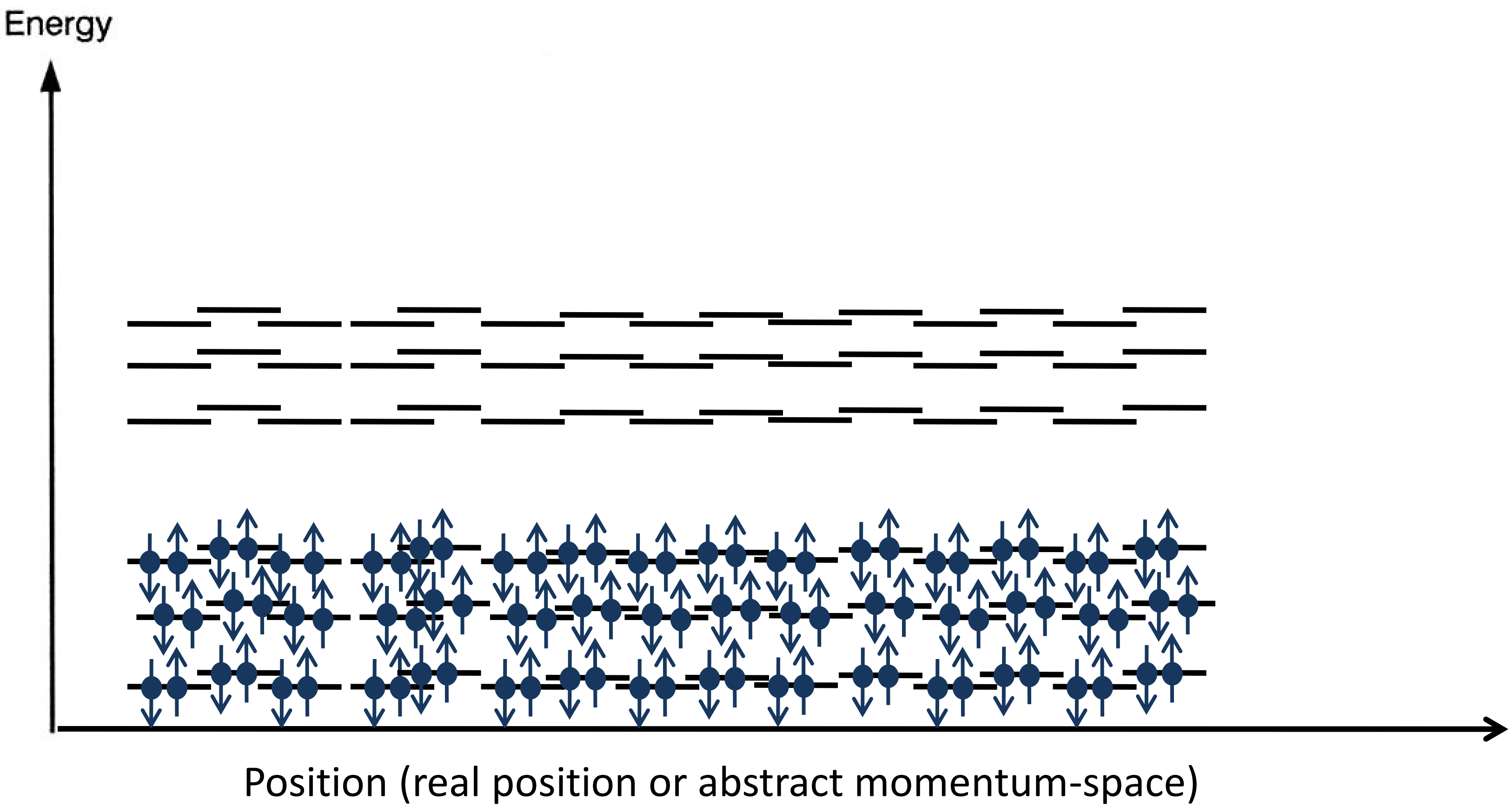
Cannot deform from torus to sphere without encountering a "singularity"

Can we deform one insulator into another
without encountering a "singularity"?

Singularity = Metal = Gap Closes

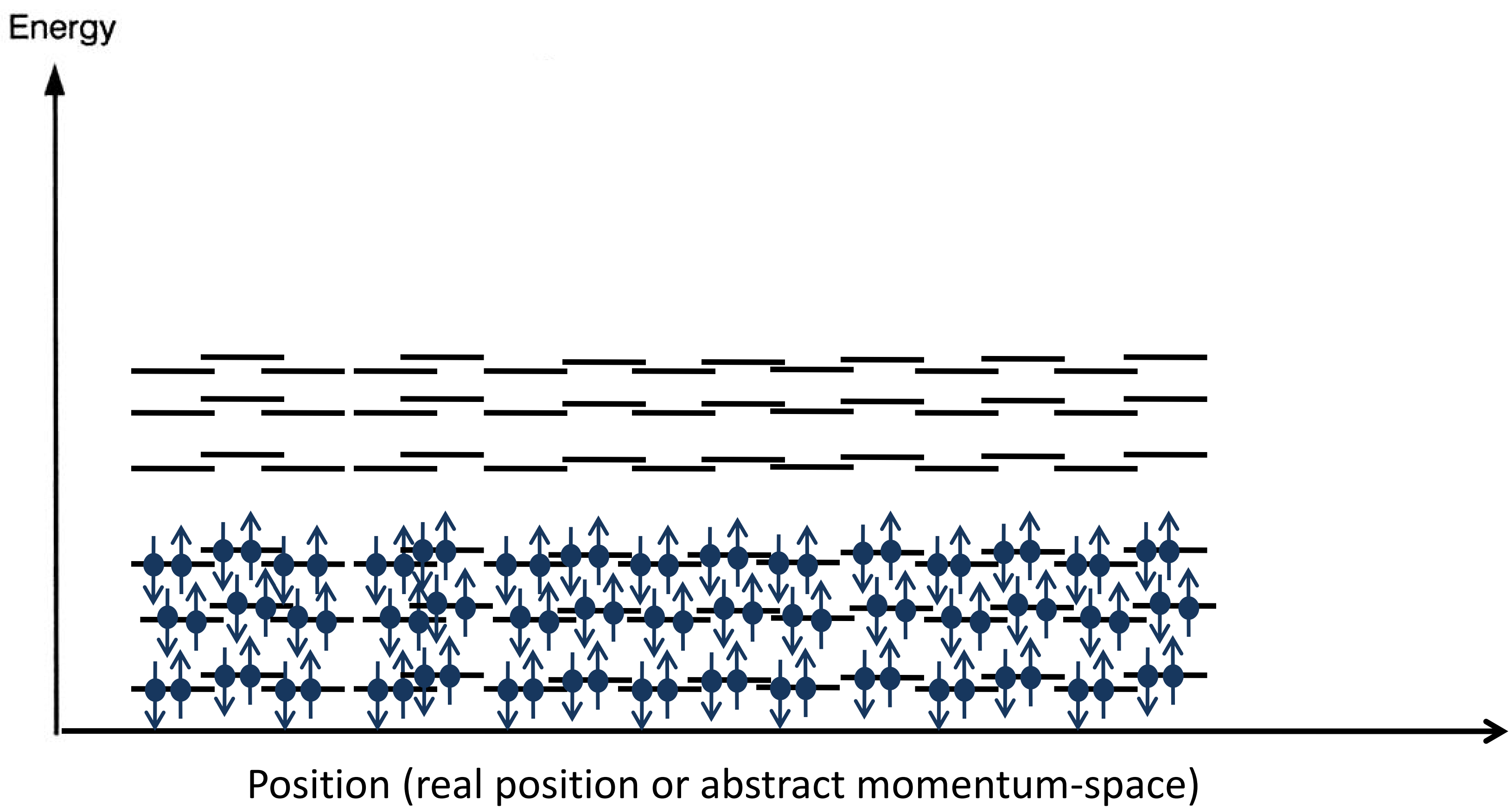
Deforming

Always an Insulator: No topological Change



Deforming

Becomes a Metal – Singularity!

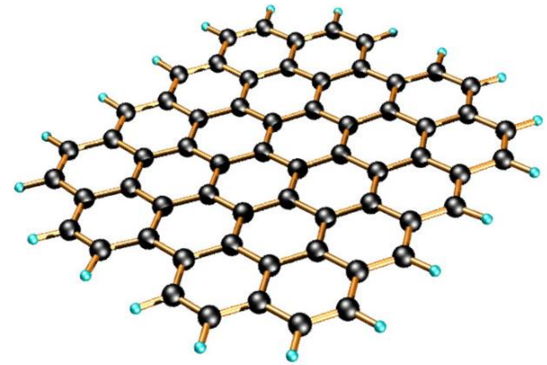


In 2 dimensions, the topological classes of insulators are indexed by an integer N

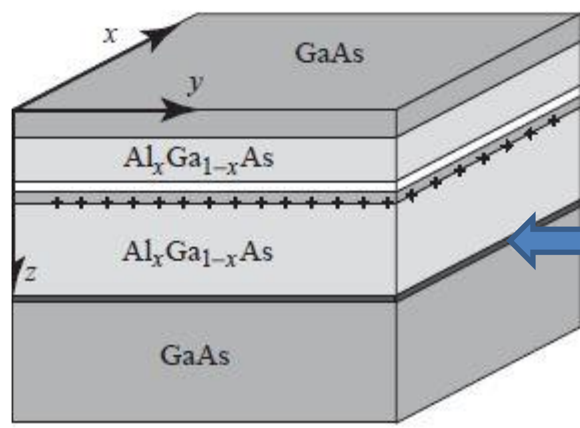
1982

David J. Thouless

Prize share: 1/2



2d Graphene



Electrons trapped here

Only interested in this 2d layer

1982

David J. Thouless

Prize share: 1/2

In 2 dimensions, the topological classes of insulators are indexed by an integer N

N=number of handles!

A precise mapping turns a wavefunction into a topological object.

To deform from one class to another you MUST deform through a metal state (singularity!).

“Regular” insulators are N=0

New Insulators are $N \neq 0$

In 2 dimensions, the topological classes of insulators are indexed by an integer N

1982

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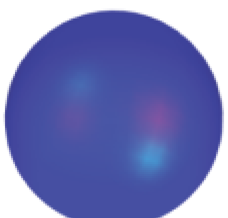
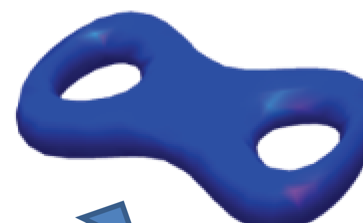
Prize share: 1/2

A gedankenexperiment:





“Regular” insulators are N=0

New Insulators are N ≠ 0

In 2 dimensions, the topological classes of insulators are indexed by an integer N

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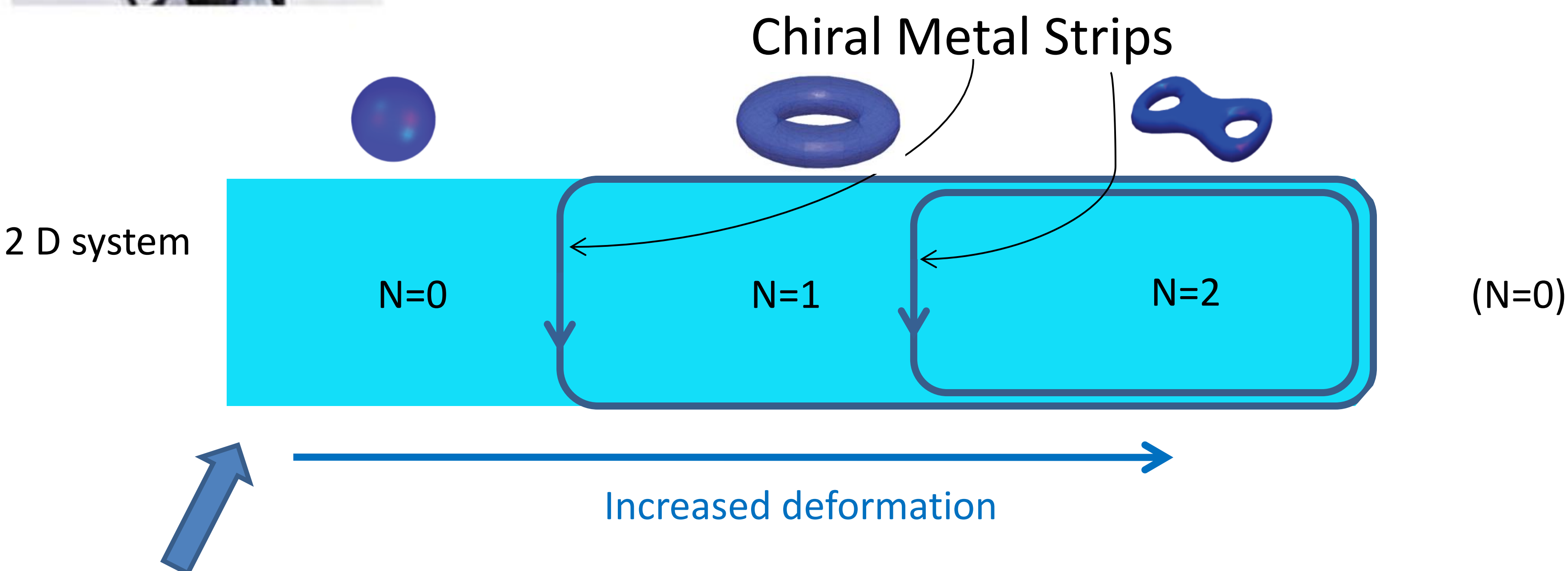


“Regular” insulators are N=0

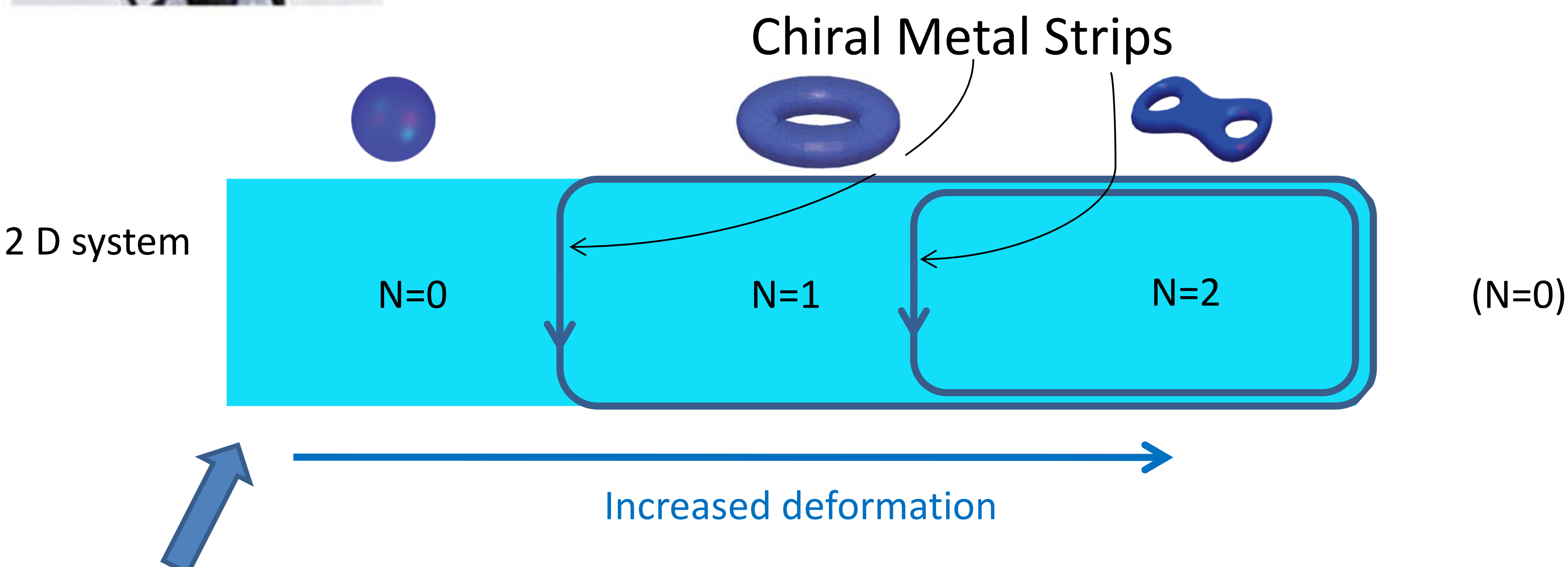
New Insulators are N ≠ 0

In 2 dimensions, the topological classes of insulators are indexed by an integer N

How to find the N≠0 insulators?

All insulators insulate in the bulk

– but N≠0 conducts along the edges



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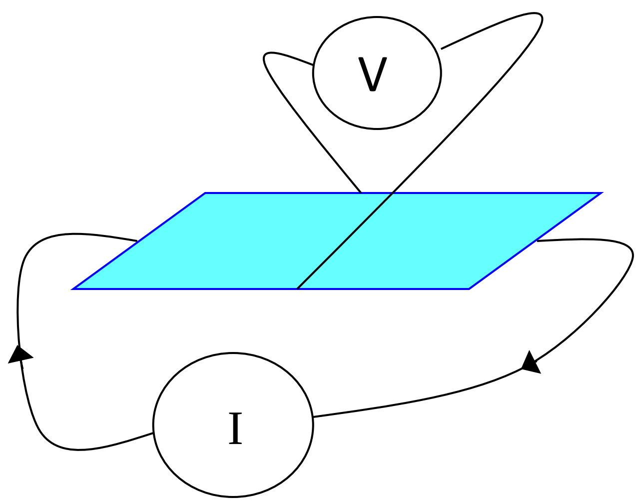
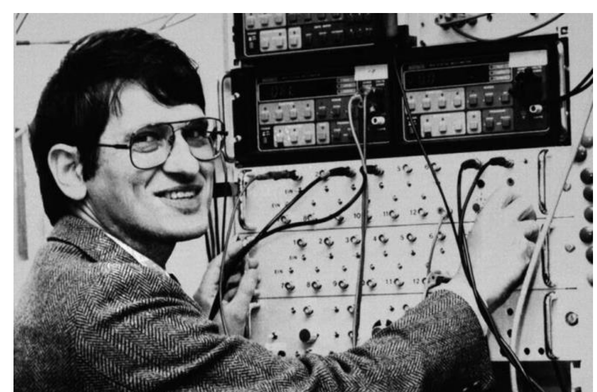


$$V = N\frac{e^2}{h}I$$

<u>Quantized Hall Effect!</u>

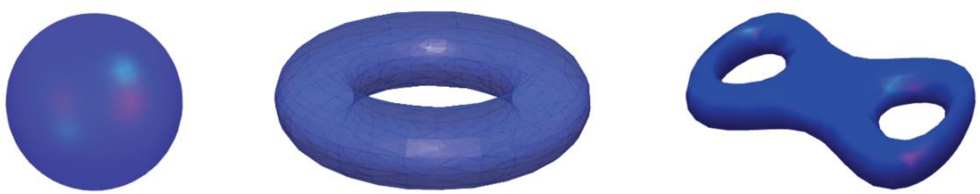
Experimental discovery
Klaus Von Klitzing, 1980
(1979 Oxford postdoc)
Nobel Prize 1985



Is this all there is?

(jump ahead to 2004)

Topology with Symmetry!



If a system has some symmetry, can consider all deformations that also preserve that symmetry.

Main example:

Insist on time reversal symmetry (no magnetism) even as we deform.

2 Classes of Time Reversal Symmetric Insulators in 2D

Normal Insulator
(no edge states)

Quantum Spin Hall Insulator
(counter propagating edge states)

Edge conductance $2\frac{e^2}{h}$

Observed in 2D HgTe Semiconductor Layers, ...

2 Classes of Time Reversal Symmetric Insulators in 3D

Normal Insulator
(no surface states)

“Topological” Insulator

Protected Surface Conductance.

What material?

These are the first few to be confirmed in experiment....

Bi_xSb_{1-x} , Bi_2Se_3 , Bi_2Te_3 , Sb_2Te_3, $(Bi,Sb)_2Te_3$, $Bi_{2-x}Sb_xTe_{3-y}Se_y$, Sb_2Te_2Se
$Bi_2(Te,Se)_2(Se,S)$, $TlBiSe_2$, $TlBiTe_2$, $TlBi(S,Se)_2$, $PbBi_2Te_4$, $PbSb_2Te_4$, $GeBi_2Te_4$
$GeBi_{4-x}Sb_xTe_7$, $(PbSe)_5(Bi_2Se_3)_6$, SnTe , $Pb_{1-x}Sn_xTe$, SmB_6 , $Bi_{15}Rh_3I_9$, Ag_2Te
PbLuSb, LuPdBi, YPtBi, LuPtBi, ...

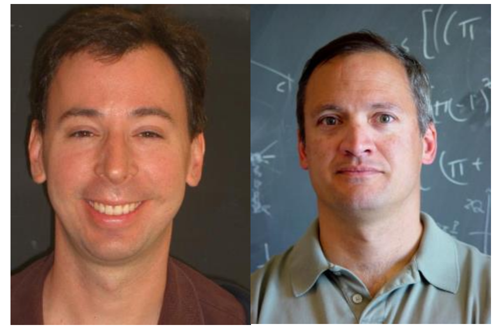
These are not unusual chemical compounds

... but no one ever noticed they were different!

(Surface conduction, magnetoelectric effect, ...)

Prediction of Topological Insulators (2006)

Liang Fu, Charlie Kane, Gene Mele



Joel Moore, Leon Balents

Rahul Roy

Grad student working with Famous physicists

Famous Physicists

Lowly grad student working by himself and being told by his supervisor to do something more useful.

Later: Oxford Postdoc, Now faculty at UCLA

Once we understood to categorize by Topology and Symmetry.....

T = Time reversal

C = Charge conjugation

Symmetry T	Symmetry C	Symmetry CT	Dimension 1	Dimension 2	Dimension 3
0	0	0	0	$\mathbb{Z}$	0
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
1	0	0	0	0	0
1	1	1	$\mathbb{Z}$	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0
1	-1	1	0	0	$\mathbb{Z}$

Quantum Hall

Sr_2RuO_4 superconductor

Quantum Spin Hall
(HgTe, InAs, ...)

He3B superfluid

3D Topological Insulator
(Lots and Lots)

Polyacetaline

InAs superconducting
wires

Periodic Table of Insulators

Is this all there is?

Once we understood to categorize by Topology and Symmetry.....

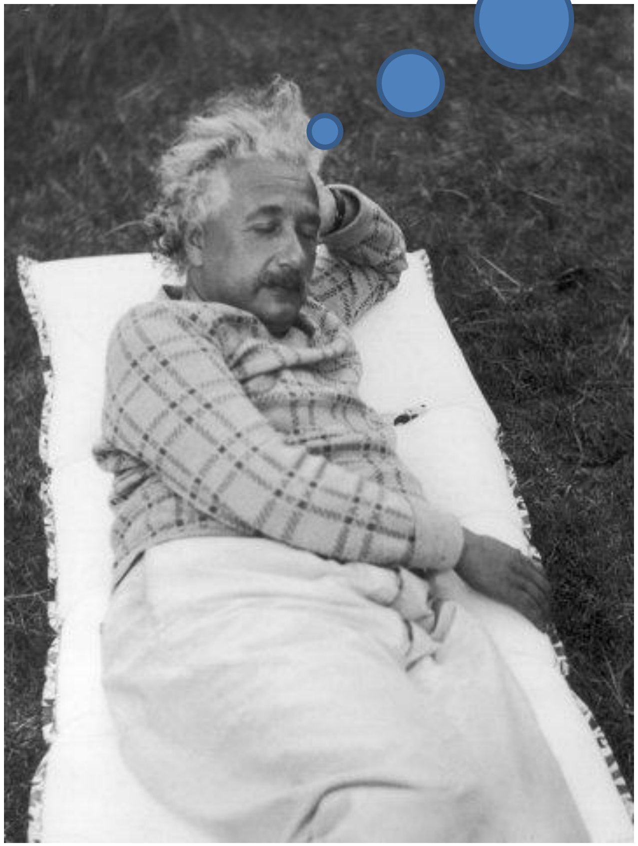
Adding a crystal reflection symmetry

Reflection	TCI/TCS	d=1	d=2	d=3
	FS1 in mirror	p=8	p=1	p=2
	FS2 in mirror	p=2	p=3	p=4
R	A	$M\mathbb{Z}$	0	$M\mathbb{Z}$
R_+	AIII	0	$M\mathbb{Z}$	0
R_-	AIII	$M\mathbb{Z} \oplus \mathbb{Z}$	0	$M\mathbb{Z} \oplus \mathbb{Z}$
R_+,R_{++}	AI	$M\mathbb{Z}$	0	0^a
	BDI	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0
	D	$M\mathbb{Z}_2^a$	$M\mathbb{Z}_2$	$M\mathbb{Z}$
	DIII	0	$M\mathbb{Z}_2^a$	$M\mathbb{Z}_2$
	AII	$2M\mathbb{Z}^a$	0	$M\mathbb{Z}_2^a$
	CII	0	$2M\mathbb{Z}^a$	0
	C	0^a	0	$2M\mathbb{Z}^a$
	CI	0	0^a	0
R_-,R_{--}	AI	0^a	0	$2M\mathbb{Z}^a$
	BDI	0	0^a	0
	D	$M\mathbb{Z}$	0	0^a
	DIII	$\mathbb{Z}_2$	$M\mathbb{Z}$	0
	AII	$T\mathbb{Z}_2^a$	$\mathbb{Z}_2$	$M\mathbb{Z}$
	CII	0	$T\mathbb{Z}_2^a$	$\mathbb{Z}_2$
	C	$2M\mathbb{Z}^a$	0	$T\mathbb{Z}_2^a$
	CI	0	$2M\mathbb{Z}^a$	0
R_{-+}	BDI, CII	$2\mathbb{Z}^a$	0	$2M\mathbb{Z}^a$
R_{+-}	DIII, CI	$2M\mathbb{Z}^a$	0	$2\mathbb{Z}^a$
R_{+-}	BDI	$M\mathbb{Z} \oplus \mathbb{Z}$	0	0^a
R_{-+}	DIII	$M\mathbb{Z}_2 \oplus \mathbb{Z}_2^a$	$M\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$M\mathbb{Z} \oplus \mathbb{Z}$
R_{+-}	CII	$2M\mathbb{Z} \oplus 2\mathbb{Z}^a$	0	$M\mathbb{Z}_2 \oplus \mathbb{Z}_2^a$
R_{-+}	CI	0^a	0	$2M\mathbb{Z} \oplus 2\mathbb{Z}^a$

\+ Adding more complicated crystal symmetries

Then move on to topologically classify metals!

THE PERIODIC TABLE OF EVERYTHING



Classification and Organization
of
All Matter

Topology
and the
Classification of Matter

New Physics Hidden in Plain Sight